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# Impedance-based Stability Analysis of Metro Traction Power System Considering Regenerative Braking

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**Abstract**—This paper presents an impedance-based stability analysis of metro traction power system considering the grid-feedback regenerative braking application. The impedance model of metro traction power system is first established based on the constant power characteristics of metro train and grid-feedback inverter. Further, the impedance-based stability criterion is developed to investigate the instability issue in traction operation and regenerative braking operation, respectively. Simulation verification is performed to validate the proposed impedance-based stability analysis method of metro traction power system. The proposed stability analysis method is able to provide design guideline for grid-feedback inverters in terms of stability.

**Keywords**— *Impedance stability analysis, Metro traction power system, Regenerative braking, Grid-feedback inverter*

## I. INTRODUCTION

The increasing power demands and traffic density in metro traction power system are promoting attentions on energy consumption reduction and system reliability improvement. Braking modes of metro train have significant effects on the energy efficiency and the power quality of metro power traction system. The existing regenerative braking (RB) modes in metro traction system mainly includes resistor braking [1], grid-feedback braking [2]-[3], hybrid braking [3], and energy storage-based regenerative braking [4]-[6]. The principle of these braking modes can be explained as following.

- Resistor braking [1]. Braking resistors are adopted to absorb the excessive power generated by braking train, which thus increases the power loss of metro traction power system.
- Grid-feedback braking [2]-[3]. Regenerative braking energy (RBE) is reversed into AC power grid by feedback converter installed in traction substation as shown in Fig.1, which is able to perform the energy conservation.
- Hybrid braking [3]. The RBE is reversed into power grid and the braking resistors are enabled to regulate catenary voltage, which is a trade-off between cost and energy efficiency.
- Energy storage (ES)-based regenerative braking [4]-[6]. The RBE is recycled in energy storage devices such as batteries or supercapacitor in braking operation, and

TABLE I COMPARISON ANALYSIS OF DIFFERENT BRAKING METHODS

Performance Indexes	Resistor [1]	Grid-feedback [2]-[3]	Hybrid [3]	ESDs [4]-[6]
Cost	low	Relatively high	medium	high
Energy efficiency	low	Relatively high	medium	high

the energy can be provided to supply the metro trains in traction operation.

A comparative analysis among different braking modes is given in Table I. Compared with other braking modes, the grid-feedback braking is commonly preferred due to the relative high energy efficiency. Fig. 1 shows the simplified diagram of metro traction power system with grid-feedback regenerative mode. The RBE can be reversed into medium voltage AC grid such as 10kV AC grid or 35kV AC grid through metro catenary. Further, the RBE can be utilized again to supply the auxiliary devices at metro station.

However, the instability issues may be caused either in traction operation or in regenerative braking operation. The oscillation issues may be caused when metro train is operated at constant power region. In [7], Routh-Hurwitz criterion is applied to analyze the mechanism of oscillation phenomenon in DC link, where the relationship between DC-link and train traction power is modelled, and an active damping method is proposed to suppress the oscillation issue. In [8], an oscillation suppression method is proposed to mitigate the DC-link oscillation phenomenon based on field-oriented vector control method. Furthermore, the instability issue may be caused in regenerative braking operation, where the grid-feedback inverters can be operated at constant power region [9]. Therefore, the instability phenomenon of metro traction power system may be caused due to the negative impedance behavior of constant power operation. However, the instability issues of metro power system with grid-feedback regenerative braking operation are slightly addressed.

Several stability analysis methods such as impedance-based analysis method [10]-[11], eigenvalue-based analysis method [12] and frequency scanning-based method [13] have been frequently developed to investigate the instability issue of grid-connected inverter system. The impedance-based method is able to identify the system stability by terminal frequency responses without using internal parameters, where

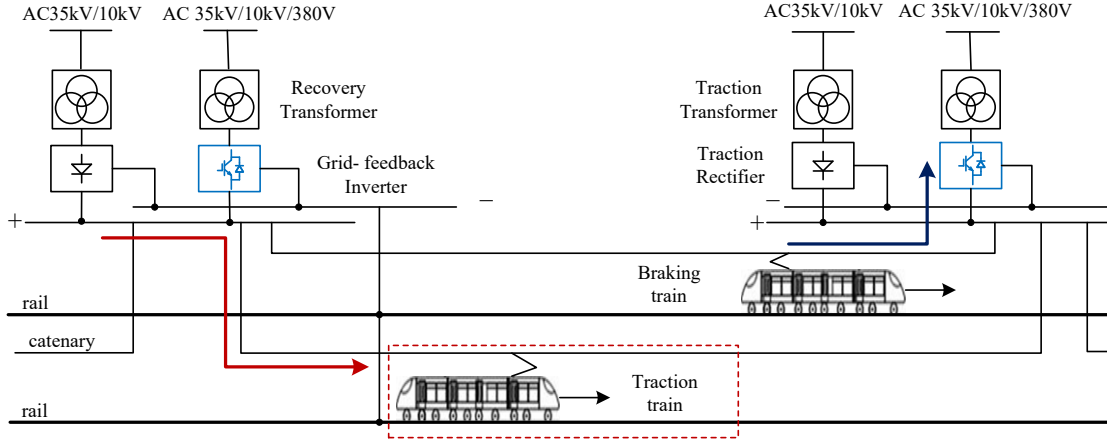


Fig. 1 The simplified diagram of metro traction power system considering grid-feedback regenerative braking operation.

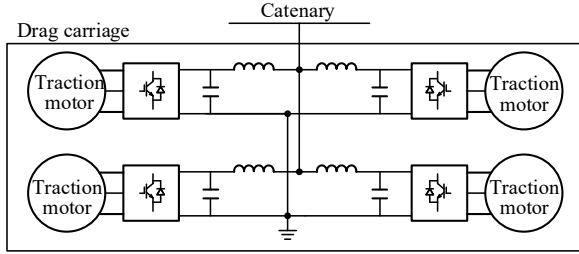


Fig. 2 The simplified diagram of one drag carriage with the four traction motors in a metro train.

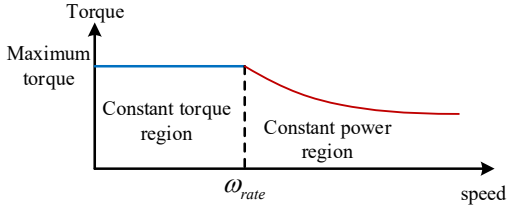


Fig. 3 The torque-speed characteristic of traction motor.

the system stability is predicted by identifying whether the ratio of inverter output impedance to equivalent system impedance meets the Nyquist stability criterion. This paper aims to develop an impedance-based stability analysis method for the metro traction power system considering grid-feedback regenerative braking.

The rest of this paper is organized as follows. In Section II, the operation principle of metro traction power system and the impedance-based stability analysis method are introduced. In Section III, the impedance model of metro traction power system is established, and the impedance-based stability analysis method is developed to investigate the effects of traction operation and regenerative braking operation on stability of metro traction power system. In Section IV, simulation is performed to validate the proposed stability analysis method. Conclusions are drawn in Section V.

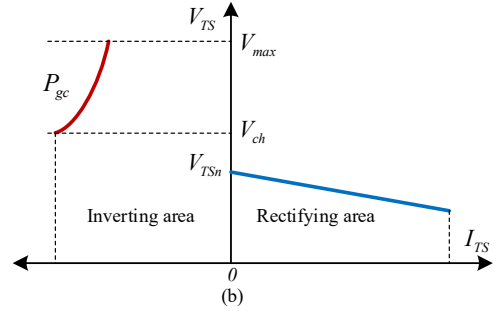
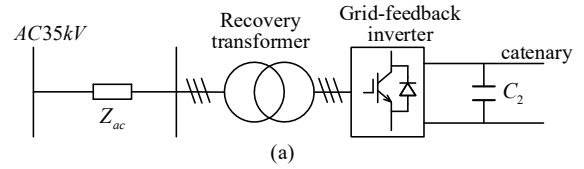


Fig. 4 The simplified diagram of grid-feedback inverter. (a) The configuration of grid-feedback inverter. (b) The operation characteristic of traction substation equipped with the grid-feedback inverter [9].

## II. SYSTEM DESCRIPTION AND METHODOLOGY

### A. System description

DC traction power system has been widely adopted as metro traction power system, which is suitable for the short-distance and multi-station transportation. Fig.1 shows the simplified diagram of metro traction power system considering grid-feedback regenerative braking, where the grid-feedback inverters are installed in traction substations. Traction power is supplied by traction transformer and rectifier from 10kV or 35kV AC grid, which is provided to supply metro trains by catenary and pantograph as shown in red line. When the metro train is operated in RB mode, the RBE can be reversed into AC grid as shown in blue line, where the grid-feedback inverter is activated to absorb RB power. Fig. 2 shows the simplified diagram of one drag carriage with four traction motors in a metro train. The bidirectional power converter can be enabled

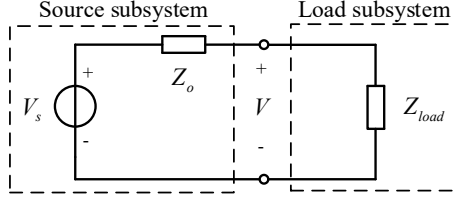


Fig. 5 The equivalent circuit of cascade system.

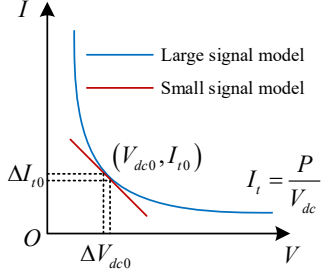


Fig. 6 The operation characteristic of active load.

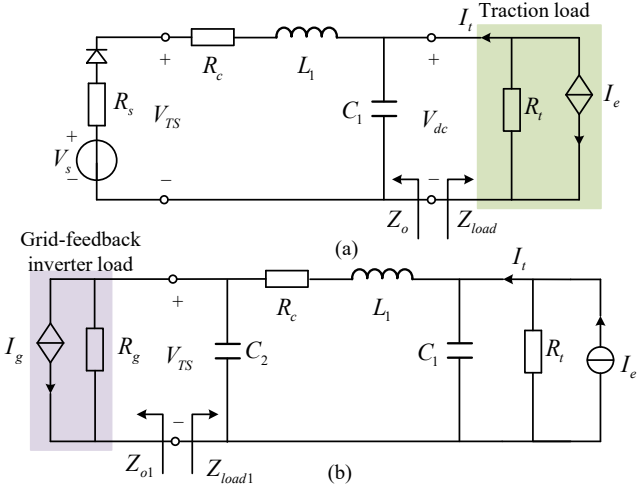


Fig. 7 The impedance model of metro traction power system. (a) Traction mode. (b) Regenerative braking mode.

TABLE II PARAMETERS OF CIRCUIT

$V_s$	$R_s$	$R_c$	$C_1$	$L_1$	$C_2$
1500V	1m $\Omega$	30m $\Omega$	32mF	0.67mH	10mF

to supply the traction power in traction operation, and feedback the RB power in braking operation.

#### 1) Operation characteristic of traction train

The induction motor is widely adopted as traction motor in practical operation. Fig. 3 shows the torque-speed characteristic of traction motor [14]-[16]. It can be seen that the traction motor is operated at the constant torque region when the practical rotational speed ( $\omega$ ) is lower than the rated rotational speed ( $\omega_{rate}$ ) of traction motor. Also, the traction motor can be operated at the constant power region when the rotational speed exceeds the rated rotational speed of traction

motor.

#### 2) Operation characteristic of grid-feedback inverter

Fig. 4 (a) shows the configuration of grid-feedback inverter where  $Z_{ac}$  represents the equivalent impedance of AC grid, and  $C_2$  represents the DC capacitor. Fig. 4 (b) shows the operation characteristic of traction substation equipped with grid-feedback inverter, where  $V_{TSn}$  represents the equivalent voltage without traction load,  $V_{ch}$  is the voltage that grid-feedback inverter can be activated.  $V_{max}$  is the maximum voltage value of grid-feedback inverter, and  $P_{gc}$  is the maximum power of grid-feedback inverter. It can be seen that the grid-feedback inverter can be operated at constant power region within the voltage range  $[V_{ch}, V_{max}]$ .

#### B. Impedance based stability analysis method

The impedance-based stability analysis method is originally proposed in [10], where the whole traction power system is partitioned into source subsystem and load subsystem as shown in Fig. 5. The source subsystem is modelled as an ideal voltage source ( $V_s$ ) with the equivalent output impedance ( $Z_o$ ) by the Thevenin equivalent principle. And the load subsystem is modelled as load impedance ( $Z_{load}$ ). The terminal voltage  $V$  at intersection point between source subsystem and load subsystem can be represented as (1).

$$V(s) = V_s(s) \cdot \frac{1}{1 + Z_o(s) / Z_{load}(s)} \quad (1)$$

With assumption that the controlled voltage source  $V_s$  and load impedance  $Z_{load}$  are internal stable, the stability of whole system can be predicted by identifying whether the ratio of output impedance and load impedance meets the Nyquist criterion [17]. The phase margin ( $PM$ ) at intersection point between output impedance ( $Z_o$ ) and load impedance ( $Z_{load}$ ) is defined as (2)

$$PM = 180^\circ - |\angle Z_o - \angle Z_{load}| \quad (2)$$

If the phase margin ( $PM$ ) is higher than 0 degree, the whole system is stable. Otherwise, the system would become unstable once the phase difference of  $Z_o$  and  $Z_{load}$  exceeds 180 degree.

### III. THE PROPOSED IMPEDANCE-BASED STABILITY ANALYSIS OF METRO POWER SYSTEM

#### A. Impedance modelling of metro train

As explained in Section II, the traction motor operated at constant power range can be modelled as an active load. Fig. 6 shows the operation characteristic of the active load [18]-[19]. For a certain steady-state operating point ( $I_{t0} = P / V_{dc0}$ ), the change rate of current can be given as (3)

$$\frac{\partial I_t}{\partial V_{dc}} = -\frac{P}{V_{dc}^2} \quad (3)$$

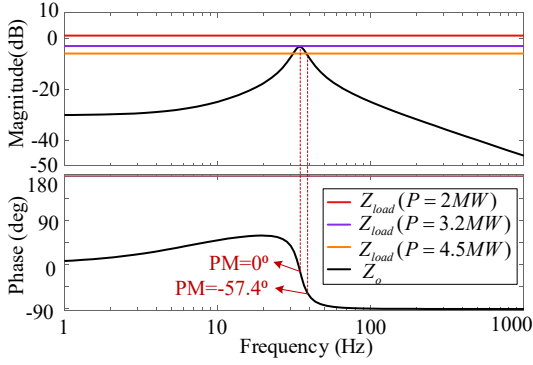


Fig. 8 The frequency response of  $Z_{load}$  and  $Z_o$  with different metro train power in traction mode.

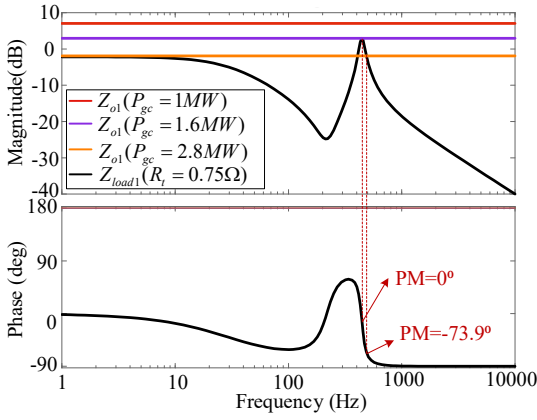


Fig. 9 The frequency response of  $Z_{load1}$  and  $Z_{o1}$  with different metro train braking power in traction mode.

The linearized model of the active load can be established at a steady-state operating point as shown in Fig. 6, which can be given as (4).

$$I_t = -\frac{P}{V_{dc0}^2} V_{dc} + 2 \frac{P}{V_{dc0}} \quad (4)$$

The (4) shows that the active load can be modelled as a current-controlled power source paralleled with a negative resistor. The value of negative resistor  $R_t$  is given as (5)

$$R_t = -\frac{V_{dc0}^2}{P} \quad (5)$$

### B. Impedance modelling of traction substation considering grid-feedback inverter

Fig. 7 (a)-(b) shows the equivalent circuit of metro traction power system equipped with the grid-feedback inverter in traction mode and braking mode, where the traditional traction substation can be modelled as an equivalent voltage source ( $V_s$ ) with equivalent resistor ( $R_s$ ). According to (4), the metro train is modelled as a current source ( $I_e$ ) paralleled with resistor ( $R_t$ )

that is calculated by (5). In this work, the grid-feedback inverter operated at constant power region is modelled as the active load. Therefore, the grid-feedback inverter can be modelled as a current source ( $I_g$ ) paralleled with resistor ( $R_g$ ). In addition,  $R_c$  represents the equivalent resistance of metro catenary.  $L_1$  and  $C_1$  represent the inductor and capacitor.  $C_2$  represents the DC capacitor of grid-feedback inverter.

### C. Impedance-based stability analysis of metro traction power system

As explained in Section II, the system stability can be predicted by identifying the impedance ratio of source subsystem and load subsystem. Fig. 7 (a)-(b) shows the impedance model of metro traction power system in traction operation and regenerative braking. For traction operation, the impedance representation of  $Z_o$  and  $Z_{load}$  can be given as (6) and (7).

$$Z_o(s) = \frac{sL_1 + R_s + R_c}{s^2 L_1 C_1 + sC_1(R_s + R_c) + 1} \quad (6)$$

$$Z_{load}(s) = R_t = -\frac{V_{dc0}^2}{P} \quad (7)$$

where  $V_{dc0}$  is the steady-state value of  $V_{dc}$ .

The system parameters are given in Table II. Fig.8 shows the frequency response of  $Z_{load}$  and  $Z_o$ . It can be seen that there exists one intersection point when the traction power ( $P$ ) is 3.2MW, where the corresponding PM is 0 degree. It means the system is critical stable. When the traction power is increased, two intersection occur, where the PM at the right intersection point is negative. It means the metro power system in traction operation would become unstable.

In regenerative braking operation, the impedance representation of  $Z_o$  and  $Z_{load}$  shown in Fig. 8 (b) can be given as (8) and (9).

$$Z_{o1}(s) = R_g = -\frac{V_{TS0}^2}{P_{gc}} \quad (8)$$

$$Z_{load1}(s) = \frac{s^2 L_1 C_1 R_t + sM + R_c + R_t}{s^3 L_1 C_1 C_2 R_t + s^2 M + sN + 1} \quad (9)$$

where  $V_{TS0}$  is the steady-state value of traction substation voltage ( $V_{TS}$ ).  $M$  and  $N$  in (9) can be given as (10)-(11).

$$M = C_2(L_1 + C_1 R_c R_t) \quad (10)$$

$$N = C_2(R_c + R_t) + C_1 R_t \quad (11)$$

Fig. 10 shows the frequency response of  $Z_{load}$  and  $Z_o$  in regenerative braking operation. It can be seen that there exists one intersection point when the RB power absorbed by grid-feedback inverter is 1.6MW, the corresponding PM at the

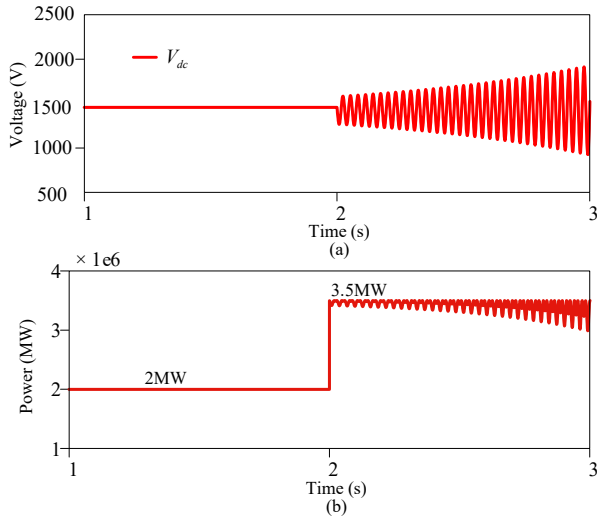


Fig. 10 The simulation results of metro power system in traction operation. (a) Catenary voltage ( $V_{dc}$ ). (b) The metro train traction power.

intersection point is 0 degree, which means the whole system is critical stable. It can be seen that there exist two intersection points when the maximum power of grid-feedback inverter is increased to 2.8MW. It can be seen that the corresponding PM at the right intersection point is -73.9 degree, which means the whole system would become unstable. The proposed stability analysis method is able to provide the optimal design guideline for grid-feedback inverter in terms of stability.

#### IV. SIMULATION VERIFICATION

To validate the proposed stability analysis method, the simulation verification is performed in Matlab/Simulink with PLECS blockset. Fig. 10 shows the simulation results of traction operation, where the Fig. 10 (a) shows the voltage of metro train ( $V_{dc}$ ), and the Fig. 10 (b) shows the traction power of metro train. It can be seen that the whole system is stable when the traction power of metro train is 2MW. Once the traction power of metro train is increased to 3.5MW, the traction power and catenary voltage would become unstable, which agrees with the theoretical analysis results in Fig. 8.

Fig. 11 shows the simulation results of regenerative braking operation, where the Fig. 11(a) shows the voltage of grid-feedback inverter ( $V_{TS}$ ) and the Fig. 11(b) shows the RB power absorbed by grid-feedback inverters. It can be seen from Fig. 11 that the whole system is stable when the maximum power of grid-feedback inverter is 1MW. Once the absorbed power of grid-feedback inverter is higher than 1.6MW, the instability phenomenon of catenary voltage and RB power can be seen as shown in Fig. 11. The simulation results agree with the theoretical analysis results as shown in Fig. 9.

#### V. CONCLUSION

This paper presents an impedance-based stability analysis method to investigate the instability issues of metro traction power system considering regenerative braking application.

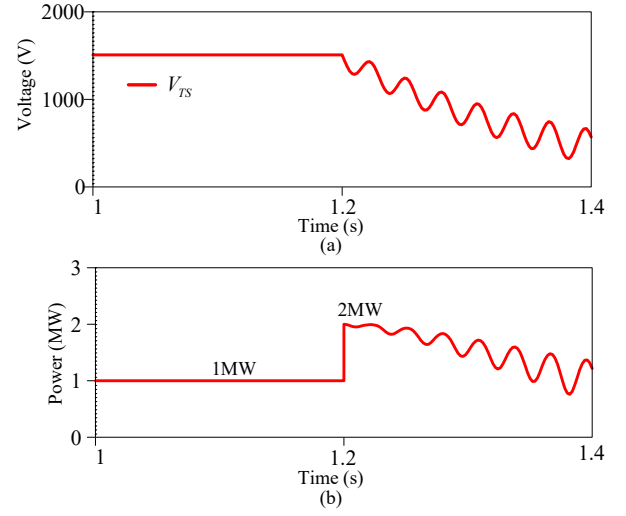


Fig. 11 The simulation results of metro power system in regenerative braking operation. (a) Catenary voltage ( $V_{TS}$ ). (b) The metro train RB power.

The impedance models of traction train and grid-feedback inverter are first established considering the constant power characteristics. And the impedance-based stability analysis method is developed to investigate the effects of traction power and regenerative braking power on system stability. The simulation results are provided to validate the proposed stability analysis method. The proposed stability analysis indicates that the instability issue can be caused when the traction trains are operated at constant power region. As the increase of traction power, the instability issues can be magnified. Furthermore, the potential instability issue can be caused when the grid-feedback inverters are operated at constant power range, where the instability issue may be caused if the regenerative braking power exceeds the power limitation. The proposed stability analysis method is able to provide design foundation for grid-feedback RB inverters in terms of stability.

#### REFERENCES

- [1] Z. Yang, H. Xia, B. Wang, and F. Lin, "An overview on braking energy regeneration technologies in Chinese urban railway transportation," in *Proc. 2047 International Power Electronics Conference and ECCE Asia (IPEC-ECCE Asia)*, pp. 2133-2139.
- [2] S. Lin, D. Huang, A. Wang, Y. Huang, L. Zhao, R. Luo, and G. Lu, "Research on the Regeneration Braking Energy Feedback System of Urban Rail Transit," *IEEE Trans. Veh. Technol.*, vol. 68, no. 8, pp. 7329-7339, Aug. 2019.
- [3] Tang P, Wang C P, and Yin X G, "Study on Inverter Based Regenerative Braking Energy Feedback and Utilization Scheme in Urban Rail Transit" *Applied Mechanics and Materials*, vol. 330, pp. 214-217, June 2013.
- [4] V. A. Kleftakis, and N. D. Hatziaargyriou, "Optimal Control of Reversible Substations and Wayside Storage Devices for Voltage Stabilization and Energy Savings in Metro Railway Networks," *IEEE Trans. Transp. Electrific.*, vol. 5, no. 2, pp. 515-523, June 2019.
- [5] Q. Qin, T. Guo, F. Lin and Z. Yang, "Energy Transfer Strategy for Urban Rail Transit Battery Energy Storage System to Reduce Peak Power of Traction Substation," *IEEE Trans. Veh. Technol.*, vol. 68, no. 12, pp. 11714-11724, Dec. 2019.

- [6] Z. Yang, Z. Yang, H. Xia, and F. Lin, "Brake Voltage Following Control of Supercapacitor-Based Energy Storage Systems in Metro Considering Train Operation State," *IEEE Trans. Ind. Electron.*, vol. 65, no. 8, pp. 6751-6761, Aug. 2018.
- [7] H. Wang, X. Ge, and Y. Liu, "An Active Damping Stabilization Scheme for the Suppression of the DC-link Oscillation in Metro Traction Drive System," *IEEE Trans. Ind. Appl.*, vol. 54, no. 5, pp. 5113-5123, Sept. 2018.
- [8] J. Hu, Z. Wang, F. Yu, and Z. Hu, "DC side Oscillation Suppression Strategy of Metro Traction Converter Based on Indirect Vector control," in *Proc. IAEAC*, 2019, pp. 2483-2487.
- [9] G. Zhang, Z. Tian, P. Tricoli, S. Hillmansen, Y. Wang, and Z. Lin, "Inverter Operating Characteristic Optimization for DC Traction Power Supply System" *IEEE Trans. Veh. Technol.*, vol. 68, no. 4, pp. 3400-3410, April 2019.
- [10] J. Sun, "Impedance-Based Stability Criterion for Grid-Connected Inverters," *IEEE Trans. Power Electron.*, vol. 26, no. 11, pp. 3075-3078, Nov. 2011.
- [11] F. Liu, J. Liu, H. Zhang and D. Xue, "Stability Issues of Z + Z Type Cascade System in Hybrid Energy Storage System (HESS)," *IEEE Trans. Power Electron.*, vol. 29, no. 11, pp. 5846-5859, Nov. 2014.
- [12] Y. Wang, X. Wang, F. Blaabjerg, and Z. Chen, "Harmonic instability assessment using state-space modeling and participation analysis in inverter-fed power systems," *IEEE Trans. Ind. Electron.*, vol. 64, no. 1, pp. 806-816, Jan. 2017.
- [13] Y. Wang, X. Wang, F. Blaabjerg, and Z. Chen, "Frequency scanning-based stability analysis method for grid-connected inverter system," in *Proc. 2017 IEEE Future Energy Electronics Conference and ECCE Asia (IFEEC-ECCE Asia)*, pp. 1575-1580.
- [14] M. Tousizadeh, H. S. Che, J. Selvaraj, N. A. Rahim, and B. Ooi, "Performance Comparison of Fault-Tolerant Three-Phase Induction Motor Drives Considering Current and Voltage Limits," *IEEE Trans. Ind. Electron.*, vol. 66, no. 4, pp. 2639-2648, April 2019.
- [15] K. Wang, Y. Li, Q. Ge, and L. Shi, "An Improved Indirect Field-Oriented Control Scheme for Linear Induction Motor Traction Drives," *IEEE Trans. Ind. Electron.*, vol. 65, no. 12, pp. 9928-9937, Dec.
- [16] P. Han, M. Cheng, and Z. Chen, "Single-Electrical-Port Control of Cascaded Doubly-Fed Induction Machine for EV/HEV Applications," *IEEE Trans. Power Electron.*, vol. 32, no. 9, pp. 7233-7243, Sept. 2017.
- [17] Y. Wang, X. Wang, F. Blaabjerg, and Z. Chen, "Harmonic resonance assessment of multiple paralleled grid-connected inverters system," in *Proc. 2017 IEEE Future Energy Electronics Conference and ECCE Asia (IFEEC-ECCE Asia)*, pp. 2070-2075.
- [18] N. Bottrell, M. Prodanovic, and T. C. Green, "Dynamic Stability of a Microgrid With an Active Load," *IEEE Trans. on Power Electron.*, vol. 28, no. 11, pp. 5107-5119, Nov. 2013.
- [19] A. M. Rahimi, and A. Emadi, "Active Damping in DC/DC Power Electronic Converters: A Novel Method to Overcome the Problems of Constant Power Loads," *IEEE Trans. on Ind. Electron.*, vol. 56, no. 5, pp. 1428-1439, May 2009.